SUPPLEMENT

Solutions of all the endogenous variables on numerical simulation

Parameter setup: $a^c = 100$, $a^o = 120$, $b^{cc} = 8$, $b^{oo} = 7$, $b^{oc} = 2$, $c^c = 8$, $c^o = 12$, $f^c = 30$, $f^o = 15$, $t^c = 0.5$, $t^o = 0.7$, $t^r = 1$

Case 1 (small substitutability):

p^c	R^c	p^o	R^o	$\frac{dp^c}{dt^o}$	$\frac{dR^c}{dt^o}$	$\frac{dp^o}{dt^o}$	$\frac{dR^o}{dt^o}$
8.54	0.573	12.8	1.14	7.79×10^{-4}	-2.32×10^{-2}	-1.07×10^{-3}	0.681

$\frac{d}{dt^o}\left(p^c + t^r + \frac{1}{2}t^c R^c\right)$	$\frac{d}{dt^o}\left(p^o + \frac{1}{2}t^o R^o\right)$
f -5.03×10^{-3}	0.807

Case 2 (large substitutability):

p^c	R^c	p^o	R^o	$\frac{dp^c}{dt^o}$	$\frac{dR^c}{dt^o}$	$\frac{dp^o}{dt^o}$	$\frac{dR^o}{dt^o}$
8.47	0.491	12.6	3.83	1.17×10^{-3}	1.02×10^{-2}	-0.841	-7.78

$\frac{d}{dt^o}\left(p^c + t^r + \frac{1}{2}t^c R^c\right)$	$\frac{d}{dt^o}\left(p^o + \frac{1}{2}t^o R^o\right)$
3.71×10^{-3}	-1.65

Note: Endogenous variables (p^c, R^c, p^o and R^o) are derived by quasi-Newton's method.

Supplement for APENDIX B:

Derivation of Property 1:

Differentiating Equation (3) with respect to R^c and R^o yields

(B.1.1) $$\Pi^c_{R^c} = 2R^c \frac{\partial p^{c*}}{\partial R^c}\left[\bar{x}^c - b^{cc}\left(p^{c*}-c^c\right)\right] + 2R^c b^{co}\left(p^{c*}-c^c\right)\frac{\partial p^{o*}}{\partial R^c} + 2\left(p^{c*}-c^c\right)x^c_R$$

and

(B.1.2) $$\Pi^c_{R^o} = 2R^c \frac{\partial p^{c*}}{\partial R^o}\left[\bar{x}^c - b^{cc}\left(p^{c*}-c^c\right)\right] + 2R^c b^{co}\left(p^{c*}-c^c\right)\left(\frac{\partial p^{o*}}{\partial R^o} + \frac{1}{2}t^o\right).$$

Note that p^{c*} and p^{o*} are the prices which satisfy profit maximization condition shown in Equation (6). The parenthesis in first term in Equation (B.1.1) and (B.1.2) is positive because deformation of Equation (A.1) yields $\bar{x}^j - b^{jj}\left(p^j - c^j\right) = \left(p^j - c^j\right)x^j_R \big/ 2t^j R^j > 0$. Third term in Equation (B.1.1) is also positive. Accordingly, proving $\partial p^{c*}\big/\partial R^c > 0$, $\partial p^{c*}\big/\partial R^o > 0$, $\partial p^{o*}\big/\partial R^c > 0$, and $\partial p^{o*}\big/\partial R^o > 0$ implies $\Pi^c_{R^c} > 0$ and $\Pi^c_{R^o} > 0$.

Deformation of total differentiation of Equation (6) with fixed R^o yields

(B.2.1) $$\frac{\partial p^{c*}}{\partial R^c} = \frac{dp^{c*}}{dR^c} = -\frac{\Pi^c_{R^c}{}'\Pi^o_{p^o}{}' - \Pi^c_{p^o}{}'\Pi^o_{R^c}{}'}{J_p} \quad \text{and} \quad \frac{\partial p^{o*}}{\partial R^c} = \frac{dp^{o*}}{dR^c} = -\frac{\Pi^c_{p^c}{}'\Pi^o_{R^c}{}' - \Pi^c_{R^c}{}'\Pi^o_{p^c}{}'}{J_p},$$

where $J_p \equiv \Pi^c_{p^c}{}'\Pi^o_{p^o}{}' - \Pi^c_{p^o}{}'\Pi^o_{p^c}{}' > 0$, shown in Equation (10), is dyanamic stability conditions for price, which is derived in (*Stage* 2: price adjustment). In the same way, deformation of total differentiation of Equation (10) with fixed R^c yields

(B.2.2) $\dfrac{\partial p^{c*}}{\partial R^o} = \dfrac{dp^{c*}}{dR^o} = -\dfrac{\Pi^{c}_{R^o}{}'\Pi^{o}_{p^o}{}' - \Pi^{c}_{p^o}{}'\Pi^{o}_{R^o}{}'}{J_p}$ and $\dfrac{\partial p^{o*}}{\partial R^o} = \dfrac{dp^{o*}}{dR^o} = -\dfrac{\Pi^{c}_{p^c}{}'\Pi^{o}_{R^o}{}' - \Pi^{c}_{R^o}{}'\Pi^{o}_{p^c}{}'}{J_p}$.

Denominators in Equation (B.2.1) and (B2.2) are positive therefore the sign of numerators of them determines the sign of $\partial p^{c*}/\partial R^c$, $\partial p^{c*}/\partial R^o$, $\partial p^{o*}/\partial R^c$, and $\partial p^{o*}/\partial R^o > 0$. Each term in the numerators in Equation (B.2.1) and (B.2.2) can be calculated as **Lemma 1**[1].

Lemma 1: $\Pi^{c}_{p^c}{}' < 0$, $\Pi^{c}_{p^o}{}' > 0$, $\Pi^{c}_{R^c}{}' > 0$, $\Pi^{c}_{R^o}{}' > 0$, $\Pi^{o}_{p^c}{}' > 0$, $\Pi^{o}_{p^o}{}' < 0$, $\Pi^{o}_{R^c}{}' > 0$, $\Pi^{o}_{R^o}{}' > 0$

Lemma 1 and dynamic stability condition $J_p \equiv \Pi^{c}_{p^c}{}'\Pi^{o}_{p^o}{}' - \Pi^{c}_{p^o}{}'\Pi^{o}_{p^c}{}' > 0$ shown in (10) specify following **Lemma 2**[2].

Lemma 2: $\dfrac{\partial p^{c*}}{\partial R^c} > 0$, $\dfrac{\partial p^{c*}}{\partial R^o} > 0$, $\dfrac{\partial p^{o*}}{\partial R^c} > 0$, $\dfrac{\partial p^{o*}}{\partial R^o} > 0$

Lemma 2 implies $\Pi^{c}_{R^c} > 0$ and $\Pi^{c}_{R^o} > 0$.

In the same manner, $\Pi^{o}_{R^c} > 0$ and $\Pi^{o}_{R^o} > 0$ can be also proved.

Derivation of Property 2:

Differentiating Equation (4) with respect to t^r yields

[1] Detailed derivation of **Lemma 1** is specified on last term in this supplement.
[2] Detailed derivation of **Lemma 2** is specified on last term in this supplement.

(B.3.1) $$\Pi^c_{t^r} = 2R^c\left(p^{c*}-c^c\right)\left(-b^{cc}+b^{co}\frac{\partial p^{o*}}{\partial t^r}\right)+2R^c\left[\overline{x}^c-b^{cc}\left(p^{c*}-c^c\right)\right]\frac{\partial p^{c*}}{\partial t^r}$$ and

(B.3.2) $$\Pi^o_{t^r} = 2R^o b^{oc}\left(p^{o*}-c^o\right)\left(1+\frac{\partial p^{c*}}{\partial t^r}\right)+2R^o\left[\overline{x}^o-b^{oo}\left(p^{o*}-c^o\right)\right]\frac{\partial p^{o*}}{\partial t^r}.$$

Note that p^{c*} and p^{o*} are the price which satisfy profit maximization condition shown in Equation (5).

Equation (B.3.1) shows the change in profit of stores in city center with the increase in t^r. First term in Equation (B.3.1) represents the change in sales, which is caused by the decrease of own demand. Second term in Equation (B.3.1) represents the change in sales, which is caused by the change in own price. Equation (B.3.2) is also interpreted in a similar way.

We determine the sign of $\partial p^{c*}/\partial t^r$ and $\partial p^{o*}/\partial t^r$ in Equation (B.3.1) and (B.3.2). Deformation of total differentiation of Equation (5) with fixed R^j yields

(B.4) $$\frac{\partial p^{c*}}{\partial t^r}=\frac{dp^{c*}}{dt^r}=-\frac{\Pi^o_{p^o}{}'\Pi^c_{t^r}{}'-\Pi^c_{p^o}{}'\Pi^o_{t^r}{}'}{J_p}$$ and $$\frac{\partial p^{o*}}{\partial t^r}=\frac{dp^{o*}}{dt^r}=-\frac{\Pi^c_{p^c}{}'\Pi^o_{t^r}{}'-\Pi^o_{p^c}{}'\Pi^c_{t^r}{}'}{J_p}.$$

Determing the sign of each term in Equation (B.4), **Lemma 3**[3] is proved.

Lemma 3: $\dfrac{\partial p^{c*}}{\partial t^r}<0$, $\dfrac{\partial p^{o*}}{\partial t^r}>0$

Lemma 3 enables us to specify the sign of Equation (B.3.1) and (B.3.2). First term in

[3] Detailed derivation of **Lemma 3** is specified on last term in this supplement.

Equation (B.3.1) is positive or negative because $\partial p^{o*} / \partial t^r > 0$. It is negative if the absolute vale of $\partial p^{o*} / \partial t^r$ is small whereas it is positive if it is sufficiently-large. While drastic change in equilibrium with transportation improvement may make the absolute value of $\partial p^{o*} / \partial t^r$ larger sufficiently, it is usually small. Our basic premise is continuous equilibrium therefore the absolute vale of $\partial p^{o*} / \partial t^r$ is small therefore first term in Equation (B.3.1) is negative. Second term in Equation (B.3.1) is negative because $\partial p^{c*} / \partial t^r < 0$ and Equation (A.1) shows $\left[\overline{x}^c - b^{cc}\left(p^{c*} - c^c\right)\right] > 0$. Accordingly $\Pi^c_{t^r} < 0$ because all terms in Equation (B.3.1) is negative. In the same manner, $\Pi^o_{t^r} > 0$ can be also proved.

Supplement for APPENDIX C:

Derivation of Property 3:

Differentiating Equation (4) with respect to t^o yields

(C.1.1) $$\Pi^c_{t^o} = R^c b^{co}\left(p^{c*} - c^c\right)\left(R^o + 2\frac{\partial p^{o*}}{\partial t^o}\right) + 2R^c\left[\overline{x}^c - b^{cc}\left(p^{c*} - c^c\right)\right]\frac{\partial p^{c*}}{\partial t^o}$$ and

(C.1.2) $$\Pi^o_{t^o} = R^o\left(p^{o*} - c^o\right)\left(-b^{oo}R^o + 2b^{oc}\frac{\partial p^{c*}}{\partial t^o}\right) + 2R^o\left[\overline{x}^o - b^{oo}\left(p^{o*} - c^o\right)\right]\frac{\partial p^{o*}}{\partial t^o}.$$

Note that p^{c*} and p^{o*} are the price which satisfy profit maximization condition shown in Equation (6).

We determine the sign of $\partial p^{c*} / \partial t^o$ and $\partial p^{o*} / \partial t^o$ in Equation (C.1.1) and (C.1.2). Deformation of total differentiation of Equation (5) with fixed R^j yields

(C.2) $$\frac{\partial p^{c*}}{\partial t^o} = \frac{dp^{c*}}{dt^o} = -\frac{\Pi^o_{p^o}{}'\Pi^c_{t^o}{}' - \Pi^c_{p^o}{}'\Pi^o_{t^o}{}'}{J_p} \quad \text{and} \quad \frac{\partial p^{o*}}{\partial t^o} = \frac{dp^{o*}}{dt^o} = -\frac{\Pi^c_{p^c}{}'\Pi^o_{t^o}{}' - \Pi^o_{p^c}{}'\Pi^c_{t^o}{}'}{J_p}.$$

Each term in numerators in Equation (C.2) can be specified. The denominator J_p is positive. As a consequence, following **Lemma 4**[4] is proved.

Lemma 4: $\frac{\partial p^{c*}}{\partial t^o} > 0$, $\frac{\partial p^{o*}}{\partial t^o}$ can be positive or negative

Lemma 4 enables us to determine the sign of Equation (C.1.1) and (C.1.2). First term in Equation (C.1.1) is positive or negative because $\partial p^{o*}/\partial t^o$ is positive or negative. It is negative if $\partial p^{o*}/\partial t^o$ is negative and its absolute value is large whereas it is positive if $\partial p^{o*}/\partial t^o$ is positive or its absolute value is sufficiently-small. As mentioned before, our basic premise is continuous equilibrium therefore the absolute vale of $\partial p^{o*}/\partial t^o$ is small so the first term in Equation (C.1.1) is positive. Second term in Equation (C.1.1) is positive because $\partial p^{c*}/\partial t^o > 0$ and (A.1) shows $\left[\overline{x}^c - b^{cc}\left(p^{c*} - c^c\right)\right] > 0$. Accordingly $\Pi^c_{t^o} > 0$ because all terms in Equation (C.1.1) is negative.

We specify the sign of Equation (C.1.2). First term in Equation (C.1.2) is positive or negative because $\partial p^{c*}/\partial t^o > 0$. However, we assume the absolute value of $\partial p^{c*}/\partial t^o$ is sufficiently-small therefore first term in Equation (C.1.2) is negative. Second term in Equation (C.1.2) is positive or negative because $\partial p^{o*}/\partial t^o$ is positive or negative. Accordingly, $\Pi^o_{t^o}$ is positive or negative because first term in Equation (C.1.2) is negative whereas second term in Equation (C.1.2) is positive or negative.

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4 Detailed derivation of **Lemma 4** is specified on last term in this supplement.

Supplement for Derivation of Lemma 1-4:

Derivation of Lemma 1 and Lemma 2

Each term in Equation (B.2.1) and (B.2.2) can be calculated as

$$
\text{(B.5)}\quad
\begin{aligned}
&\Pi^c_{p^c}{}' = -\frac{7}{2}b^{cc}R^c - \frac{1}{t^c}\left[\overline{x}^c - \frac{1}{2}b^{cc}\left(p^c - c^c\right)\right], \quad \Pi^c_{p^o}{}' = \frac{3}{2}b^{co}\left(R^c - \frac{p^c - c^c}{2t^c}\right),\\
&\Pi^c_{R^c}{}' = 2x^c_R - b^{cc}\left(p^c - c^c\right), \quad \Pi^c_{R^o}{}' = b^{co}t^o\left(R^c - \frac{p^c - c^c}{2t^c}\right),\\
&\Pi^o_{p^c}{}' = \frac{3}{2}b^{oc}\left(R^o - \frac{p^o - c^o}{2t^o}\right), \quad \Pi^o_{p^o}{}' = -\frac{7}{2}b^{oo}R^o - \frac{1}{t^o}\left[\overline{x}^o - \frac{1}{2}b^{oo}\left(p^o - c^o\right)\right],\\
&\Pi^o_{R^c}{}' = b^{oc}t^c\left(R^o - \frac{p^o - c^o}{2t^o}\right), \quad \text{and} \quad \Pi^o_{R^o}{}' = 2x^o_R - b^{oo}\left(p^o - c^o\right).
\end{aligned}
$$

On the *PML* shown in Figure 2, $p^j - c^j = 0$ at $R^j = 0$. It shows such situation that if there are countless stores throughout the ring road, each store conforms marginal cost pricing because such situation is equivalent to perfect competition. Therefore, substituting $R^j = 0$ and $p^j - c^j = 0$ into Equation (A.3) yields $dp^j / dR^j = 2t^j$, which expresses the slope of *PML* at $R^j = 0$. At the arbitrary point $\left(R^j, p^j - c^j\right)$ on the *PML*, the slope of line which connects origin and the arbitrary point is smaller than $2t^j$ because *PML* is concave function. This condition is shown as $\left(p^j - c^j\right)/R^j < 2t^j$, which is transformed as

$R^j - \left(p^j - c^j\right)/2t^j > 0$.

The sign of each term shown in Equation (B.5) is determinable from above condition

$R^j - \left(p^j - c^j\right)\Big/2t^j > 0$, $\bar{x}^j - b^{jj}\left(p^j - c^j\right)\Big/2 > 0$ which is derived in Appendix A, and $2x_R^j - b^{jj}\left(p^j - c^j\right) > 0$ which is assumed in Section 2. They are notified as following **Lemma 1**.

Lemma 1: $\Pi^c_{p^c}{}' < 0$, $\Pi^c_{p^o}{}' > 0$, $\Pi^c_{R^c}{}' > 0$, $\Pi^c_{R^o}{}' > 0$, $\Pi^o_{p^c}{}' > 0$, $\Pi^o_{p^o}{}' < 0$, $\Pi^o_{R^c}{}' > 0$, $\Pi^o_{R^o}{}' > 0$

Lemma 1 proves that numerator of Equation (B.2.1) and (B.2.2) is negative. On the other hand, dynamic stability condition shown in Equation (10) specifies that denominators in Equation (B.2.1) and (B.2.2) are positive. Therefore, following **Lemma 2** is proved.

Lemma 2: $\dfrac{\partial p^{c*}}{\partial R^c} > 0$, $\dfrac{\partial p^{c*}}{\partial R^o} > 0$, $\dfrac{\partial p^{o*}}{\partial R^c} > 0$, $\dfrac{\partial p^{o*}}{\partial R^o} > 0$

Derivation of Lemma 3

Dynamic stability condition shown in Equation (10) proves that the denominators in Equation (B.2.1) and (B.2.1) are positive. Partially differentiating Equation (6) with respect to t^r yields

(B.6) $$\Pi^c_{t^r}{}' = -2b^{cc}\left(R^c - \frac{p^c - c^c}{2t^c}\right) \text{ and } \Pi^o_{t^r}{}' = -2b^{oc}\left(R^o - \frac{p^o - c^o}{2t^o}\right).$$

The numerators in Equation (C.2) are expressed as following by substituting Equation (B.5) and (B.6).

(B.7.1)

$$\Pi^{o}_{p^o}{}'\Pi^{c}_{t^r}{}' - \Pi^{c}_{p^o}{}'\Pi^{o}_{t^r}{}'$$
$$= 2\left(R^c - \frac{p^c - c^c}{2t^c}\right)\left[\left(\frac{7}{2}b^{cc}b^{oo} - \frac{3}{2}b^{co}b^{oc}\right)\left(R^o - \frac{p^o - c^o}{2t^o}\right) + \frac{5}{2}b^{cc}b^{oo}\frac{p^o - c^o}{2t^o} + \frac{b^{cc}}{t^o}\overline{x}^o\right]$$

(B.7.2) $$\Pi^{c}_{p^c}{}'\Pi^{o}_{t^r}{}' - \Pi^{o}_{p^c}{}'\Pi^{c}_{t^r}{}' = -2b^{oc}\left(R^o - \frac{p^o - c^o}{2t^o}\right)\left(2b^{cc}R^o + b^{cc}\frac{p^c - c^c}{4t^c} + \frac{\overline{x}^c}{t^c}\right)$$

Equation (B.7.2) is negative therefore $\partial p^{o*}/\partial t^r > 0$. To specify the sign of Equation (B.7.1), we focus on the characteristics of demand function shown in Equation (1) and (2). Usual good's elasticity of own price is greater than absolute value of elasticity of substitute good's price. If they are equal, they are perfect substitute goods. This characteristics correspond to such condition that $b^{cc} \geq b^{co}$ and $b^{oo} \geq b^{oc}$. This condition yields that Equation (B.7.1) is positive therefore $\partial p^{c*}/\partial t^r < 0$.

Lemma 3: $\dfrac{\partial p^{c*}}{\partial t^r} < 0$, $\dfrac{\partial p^{o*}}{\partial t^r} > 0$

Derivation of Lemma 4

Dynamic stability condition shown in Equation (10) proves that denominator of Equation (C.2) is positive. Partially differentiating Equation (6) with respect to t^o yields

(C.3) $$\Pi^{c}_{t^o}{}' = b^{co}R^o\left(R^c - \frac{p^c - c^c}{2t^c}\right) \text{ and } \Pi^{o}_{t^o}{}' = -b^{oo}R^o\left(R^o - \frac{p^o - c^o}{2t^o}\right) + b^{oo}R^o\frac{p^o - c^o}{2t^o}.$$

The numerators in Equation (C.2) are expressed as following by substituting Equation (B.5) and (C.3).

(C.4.1) $$\Pi^{o}_{p^o}{}'\Pi^{c}_{t^o}{}' - \Pi^{c}_{p^o}{}'\Pi^{o}_{t^o}{}' = -b^{co}R^{o}\left(R^{c} - \frac{p^{c}-c^{c}}{2t^{c}}\right)\left(2b^{oo}R^{o} + b^{oo}\frac{p^{o}-c^{o}}{t^{o}} + \frac{\overline{x}^{o}}{t^{o}}\right)$$

(C.4.2)

$$\begin{aligned}
&\Pi^{c}_{p^c}{}'\Pi^{o}_{t^o}{}' - \Pi^{o}_{p^c}{}'\Pi^{c}_{t^o}{}' \\
&= R^{o}\left(R^{o} - \frac{p^{o}-c^{o}}{2t^{o}}\right)\left[\left(\frac{7}{2}b^{cc}b^{oo} - \frac{3}{2}b^{co}b^{oc}\right)\left(R^{c} - \frac{p^{c}-c^{c}}{2t^{c}}\right) + \frac{5}{2}b^{cc}b^{oo}\frac{p^{c}-c^{c}}{2t^{c}} + \frac{b^{oo}}{t^{o}}\overline{x}^{c}\right] \\
&\quad - b^{oo}R^{o}\frac{p^{o}-c^{o}}{2t^{o}}\left[b^{cc}\left(R^{c} - \frac{p^{c}-c^{c}}{2t^{c}}\right) + \frac{5}{2}b^{cc}R^{c} + \frac{\overline{x}^{c}}{t^{c}}\right]
\end{aligned}$$

Equation (C.4.1) is negative therefore $\partial p^{c*} / \partial t^{o} > 0$. To specify the sign of Equation (C.4.2), we assume the usual characteristics of demand function $b^{cc} \geq b^{co}$ and $b^{oo} \geq b^{oc}$ which is shown in previous section. On this characteristics, first term in Equation (C.4.2) is positive whereas second term in Equation (C.4.2) is negative. Accordingly, Equation (C.4.2) is positive or negative therefore $\partial p^{o*} / \partial t^{o}$ is also positive or negative.

Lemma 4: $\dfrac{\partial p^{c*}}{\partial t^{o}} > 0$, $\dfrac{\partial p^{o*}}{\partial t^{o}}$ is positive or negative